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Finite size phase transitions in QCD with adjoint fermions

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Abstract: We perform a lattice investigation of QCD with three colors and 2 flavors of Dirac (staggered) fermions in the adjoint representation, defined on a $4d$ space with one spatial dimension compactified, and study the phase structure of the theory as a function of the size L_c of the compactified dimension. We show that four different phases take place, corresponding to different realizations of center symmetry: two center symmetric phases, for large or small values of L_c, separated by two phases in which center symmetry is broken in two different ways; the dependence of these results on the quark mass is discussed. We study also chiral properties and how they are affected by the different realizations of center symmetry; chiral symmetry, in particular, stays spontaneously broken at the phase transitions and may be restored at much lower values of the compactification radius. Our results could be relevant to a recently proposed conjecture [7] of volume indepedence of QCD with adjoint fermions in the large N_c limit.

Keywords: Lattice Gauge Field Theories, Confinement

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Contents

1 Introduction

Extracting the physics of large N_c QCD from simulations or analytic computations performed on small volumes, down to a single site, has been the aim of large theoretical efforts since the original work by Eguchi and Kawai [1]. However, the expectation that physics on a d-dimensional $L_1 \times \ldots L_d$ periodic lattice should be independent, as $N_c \to \infty$, of the lattice sizes $L_1, \ldots L_d$, is justified if no transition occurs as the sizes $L_1, \ldots L_d$ are changed: this is not true in ordinary Yang-Mills theories in $d > 2$, which undergo phase transitions as one or more dimensions are compactified below a given threshold, corresponding to the spontaneous breaking of the center symmetry in that direction. One or more Wilson (Polyakov) lines in the compactified directions get a non-zero expectation value, i.e. one or more deconfining-like transitions take place (see ref. [2] for a review on the subject).

Large N_c QCD with N_f fermions in the antisymmetric representation, or QCD(AS), has attracted lot of theoretical interest in the last few years, as a different $N_c \to \infty$ limit of ordinary QCD: it is indeed equivalent to ordinary QCD for $N_c = 3$. Orientifold planar equivalence states that QCD(AS) is equivalent, in the large N_c limit and in the charge-even sector of the theory, to QCD with fermions in the adjoint representation, QCD(Adj) [3]. Such equivalence is guaranteed if the charge conjugation symmetry is not spontaneously broken in QCD(AS) [4]: that is true for sufficiently large volumes, but fails in presence of a compactified dimension, below a typical compactification radius [5, 6].

Recently it has been made the hypothesis [7] that instead QCD with one or more adjoint fermions would persist in the confined phase (i.e. no phase transition would occur) as one of the dimensions is compactified, if the boundary conditions for fermions are periodic in that direction (i.e. if the compactified dimension is not temperature-like). That would imply physics be volume independent in the large N_c limit, and thus would open the possibility of studying QCD(Adj) down to arbitrarily small volumes, finally connecting information, via orientifold planar equivalence, to the physical properties of QCD(AS) on large volumes.

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That would be a breakthrough in the study of large N_c QCD, with possible important implications also for the knowledge of real QCD. It is therefore of great importance to work on such conjecture, providing evidence from numerical lattice simulations.

The argument given in ref. [7] is based on the analysis of the 1-loop effective potential for the Polyakov (Wilson) line, Ω, taken along a compactified dimension of length L_c, in presence of N_f ($N_f/2$) flavors of massless Majorana (Dirac) adjoint fermions, which is given, up to a constant, by

$$V_{\text{eff}}(\Omega) = \frac{1 \mp N_f}{24\pi^2 L_c^4} \sum_{i,j=1}^{N_c} [\phi_i - \phi_j]^2([\phi_i - \phi_j] - 2\pi)^2 \tag{1.1}$$

where the $-/+$ sign occurs for periodic/antiperiodic boundary conditions (p.b.c./a.b.c.) in the given direction, $e^{i\phi_j}$, $j = 1, \dots N_c$ are the eigenvalues of the Polyakov line, which satisfy the constraint $\prod_j e^{i\phi_j} = 1$, and $[x] = (x \bmod 2\pi)$.

At arbitrarily small compactified dimension, the 1-loop potential gives all relevant information. For $N_f = 0$ or for a.b.c., the potential implies attraction among the eigenvalues: indeed it is minimized when they coincide, meaning that the Polyakov line alignes itself along some center element and center symmetry is spontaneously broken. Instead, for $N_f > 1$ and p.b.c. the potential changes sign and becomes repulsive: the Polyakov line is disordered in this case and center symmetry is not broken, exactly as in the infinite volume, confined phase. The potential in eq. (1.1) is given for zero quark mass m and corrections are expected for finite m, however results should not change as finite quark masses are switched on, and indeed it has been shown in ref. [8] that the disordered confined phase is always preferred for small enough (compared to Λ_{QCD}) quark masses. Analogous results are obtained when studying the 1-loop potential in the lattice regularized theory [11], while contrasting results (i.e. a center symmetry breaking 1-loop potential) obtained in a three-dimensional reduced model [12] should be interpreted with care and may be due to the absence of certain center stabilizing relevant operators which should in principle be included in the reduced model [11]. Finally, a similar effect is obtained in the pure SU(N_c) gauge theory by adding a trace deformation (i.e. proportional to the adjoint trace of the Polyakov line) to the usual Yang-Mills action [9, 10].

An essential requirement for volume independence in the large N_c limit is that nothing, i.e. no phase transition, happens as the compactified dimension spans from infinity to the arbitrarily small, perturbative regime, i.e. that the system stays always confined and does not explore any other phase in between. That can be verified by numerical lattice simulations. While numerical results exist for pure SU(N_c) gauge theories with trace deformations [10], no result still exists concerning QCD with dynamical adjoint fermions.

In the present paper we consider the example of $N_c = 3$ with 2 flavors of Dirac adjoint fermions, providing numerical evidence that the requirement above is highly non-trivial. The system that we have studied undergoes several phase transitions when shrinking from the large volume to the small volume confined regimes. Our study has been performed for finite values of the quark masses: the fact that these phase transitions may disappear in the chiral limit is not excluded, but our present data show that this is non-trivial.

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The $N_c = 3$ case may in principle be quite far from the large N_c limit and one may question about the relevance of our results to the conjecture proposed in ref. [7]. However the argument for the absence of center symmetry breaking in the perturbative, small L_c limit, which is based on the analysis of the 1-loop potential in eq. (1.1), is valid independently of N_c. Therefore the fact that different phases may be present between the large volume and the small volume regime, as established for $N_c = 3$ in the present paper, is a possibility that should be taken into account also for larger values of N_c. To summarize, while our findings could be relevant just to the particular case studied here and to the discretization setup used, they are a warning claiming for further studies with different values of N_c and approaching the continuum limit. A step in this last direction is taken already in our investigation: some of our data have been obtained for different values of the lattice spacing, showing that at least the first critical length which is met when shrinking the compactified dimension may have a well defined physical value in the continuum limit, i.e. that it has the correct behavior expected by Renormalization Group arguments.

The paper is organized as follows. In section II we present details about lattice QCD with adjoint fermions and about our numerical simulations. In section III we present our numerical results, concerning the realization of both center symmetry and chiral symmetry as a function of the size of the compactified dimension, and discuss the limit of zero quark mass and the continuum limit as well. In section IV we discuss our conclusions.

2 QCD(Adj) on the lattice and simulation details

Fermions in the adjoint representation of SU(3) have 8 color degrees of freedom and can be described by 3×3 hermitian traceless matrices:

$$Q = Q^a \lambda_a \tag{2.1}$$

where λ_a are the Gell-Mann's matrixes. The elementary parallel trasports acting on them are given by the 8−dimensional $U_{(8)}$ representation (which is real) of the gauge links

$$U^{ab}_{(8)\ i,\mu} = \frac{1}{2}\mathrm{Tr}\left(\lambda^a U_{(3)\ i,\mu} \lambda^b U^{\dagger}_{(3)\ i,\mu}\right) \tag{2.2}$$

where i is a lattice site. The full discretized lattice action used in our investigation is given by

$$S = S_G\left[U_{(3)}\right] + \sum_{i,j,a,b} \bar{Q}^a_i M\left[U_{(8)}\right]^{a,b}_{i,j} Q^b_j \tag{2.3}$$

S_G is the standard pure gauge plaquette action with links in the 3-dimensional representation

$$S_G = \beta \sum_{\Box}\left(1 - \frac{1}{N_c}\mathrm{Tr}\Pi_{\Box}\right)$$

where β is the inverse gauge coupling $\beta = 2N_c/g_0^2$ and the sum is over all elementary plaquette operators $\Pi_{\Box}$ of the lattice. M is the fermionic matrix, which has been chosen according to the standard staggered fermion discretization:

$$M\left[U_{(8)}\right]_{i,j}^{a,b} = am\delta_{i,j}\delta^{a,b} + \frac{1}{2}\sum_{\nu=1}^{4}\eta_{i,\nu}\left(U_{(8)\ i,\nu}^{a,b}\delta_{i,j-\hat{\nu}} - U_{(8)\ i-\hat{\nu},\nu}^{\dagger\ a,b}\delta_{i,j+\hat{\nu}}\right) \tag{2.4}$$

Notice that, since gauge links acting on adjoint fermions are real matrixes, $M_{i,j}^{a,b}$ is real as well. The functional integral, describing 2 flavors of Dirac adjoint fermions in the continuum limit, is then given by

$$Z = \int \mathcal{D}U e^{-S_G}(\det M[U_{(8)}])^{1/2} \tag{2.5}$$

Due to the reality of M, that can be expressed in terms of a real pseudofermion field living only on even sites of the lattice as follows

$$Z = \int \mathcal{D}U\mathcal{D}\Phi_e e^{-S_G}\exp\left(-\Phi_e^t(M^tM)_{ee}^{-1}\Phi_e\right) \tag{2.6}$$

In this form the functional integral distribution can be easily sampled by a standard Hybrid Monte Carlo algorithm. In particular we have adopted the so-called Φ algorithm [13].

We shall consider in the following a system in which one dimension is compactified and has size L_c. Adjoint parallel transports are blind to the center of the gauge group, hence the presence of adjoint fermions does not break explicitely center symmetry in the compactified direction. The fundamental Polyakov line $L_{(3)}$ taken along the compactified dimension is not invariant under the same symmetry, and is therefore an exact order parameter for its spontaneous breaking.

If fermions are given a.b.c. in the compactified dimension, the Feynman path integral can be interpreted as the partition function at temperature $T = 1/L_c$. The spontaneous breaking of center symmetry is then associated to deconfinement. Various numerical studies of finite temperature QCD(Adj) have been performed in the past, mostly aimed at studying the relation between deconfinement and chiral symmetry restoration [14–16].

In our investigation we have considered fermions with p.b.c. along the compactified dimension. This theory cannot be interpreted as a thermodynamical system (apart from the case of infinite fermion mass), but must considered simply as a system with a shorter, compactified spatial dimension. Nevertheless we shall refer to the phase where the fundamental Polyakov line acquires a non-zero expectation value, oriented along an element of the center of the gauge group, as a "deconfined" phase.

In most of our numerical simulations we have kept a fixed number of lattice sites along the compactified direction, and varied its physical size by tuning the lattice spacing a via the inverse gauge coupling β. Asymptotic freedom implies that $a \to 0$ as $\beta \to \infty$, hence the compactified dimension is made shorter and shorter as β is increased. Since we are mostly interested in studying which phases are explored by the system as the size of the compactified dimension is changed from large to small values, and not in the exact location of the possible phase transitions, it is enough for our purposes to look at the phase structure of theory as a function of β at fixed number of lattice sites. However we shall try to get an estimate of physical scale ratios, by means of the 2-loop perturbative β-function, describing the asymptotic dependence of the lattice spacing a on β, or alternatively by performing numerical simulations also at a fixed value of the lattice spacing, in which L_c is tuned by changing the number of lattice sites in the compactified dimension.

It is essential to repeat our investigation for different values of the quark mass. An infinite quark mass corresponds to the pure gauge theory, where it is well known that two different phases, deconfined or confined, take place respectively for short or large values of the compactified dimension; such phases are separated by a first order transition [17]. For small enough quark masses one expects instead center symmetric phases both for a very large or a very short size of the compactified dimension, with or without different phases in between. It is then clear that a highly non-trivial phase structure must be found in the quark mass - L_c plane. For this reason we have performed numerical simulations at five different values of the bare quark mass, $am = 0.50$, 0.10, 0.05, 0.02 and 0.01.

In all cases we have chosen molecular dynamics trajectories of length 0.5, with typical integration steps δt ranging from 0.025 (for $am = 0.50$) to 0.005 (for $am = 0.01$), so as to keep an acceptance rate around 80%. For each parameter choice (am, β, L_c) we have collected between 2k and 20k trajectories. Numerical simulations have been performed on the APEmille machine in Pisa and the apeNEXT facility in Rome.

3 Numerical results

We start by presenting results obtained on a lattice $16^3 \times \hat{L}_c$, with $\hat{L}_c = 4$ and p.b.c. for all fields. The shorter direction plays the role of the compactified dimension of length $L_c = \hat{L}_c a(\beta, m)$. In order to distinguish the various phases corresponding to different possible realizations of center symmetry, we have studied the expectation value and distribution of the trace of the Polyakov line, $L_{(3)}$, taken along the shorter direction and averaged over the orthogonal $3d$ space, as a function of β and am.

In figures 1 and 2 we show scatter plots of the distribution of $L_{(3)}$ in the complex plane, for $am = 0.1$ and $am = 0.02$ respectively, and for different values of β. In each case four different phases are clearly distinguishable:

- In a first phase at low β values, correponding to large values of L_c, $L_{(3)}$ is distributed around zero with a vanishing expectation value. That corresponds to the usual confined, center symmetric phase of QCD(Adj).

- As β increases (L_c is made shorter), the system passes into a phase with a broken center symmetry, with $L_{(3)}$ aligned along one of the center elements of SU(3). We call this phase *deconfined*, since it is analogous to the usual high temperature phase of QCD.

- At very high values of β (very small L_c) the system is again in a center symmetric, confined-like phase, in agreement with the prediction of the one-loop potential in eq. (1.1). Since this phase seems to be distinguished and separated from the low temperature confined phase, we refer to it as *re-confined*.

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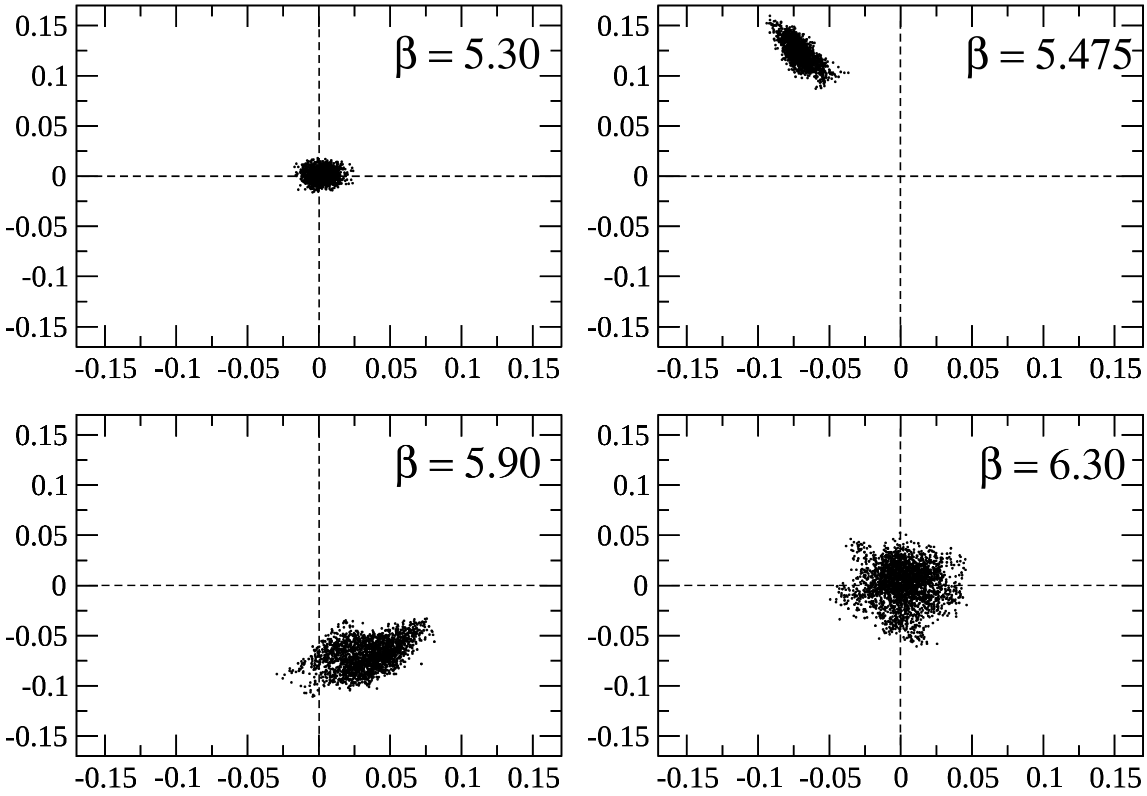

Figure 1. Scatter plots of the distribution of the trace of the Polyakov line, $L_{(3)}$, in the complex plane, for various values of β and at a fixed quark mass $am = 0.10$ on a $16^3 \times 4$ lattice.

- Finally, a new phase is present for values of β (L_c) between the deconfined and the re-confined phase, in which center symmetry is broken by a non-zero expectation value of $L_{(3)}$, which is however oriented along directions rotated roughly by an angle π, in the complex plane, with respect to those corresponding to center elements. This phase is completely analogous to that found by the authors of ref. [10] in their study of pure gauge SU(N_c) gauge theories with adjoint Polyakov line deformations, and predicted again by the same authors, on the basis of the 1-loop effective potential, also in the case of QCD(Adj) with p.b.c. and for some range of m and L_c [8]. We shall refer to this phase as a *split* or *skewed* phase, following the convention in ref. [10].

The transitions among the different four phases are also clear from figure 3, where we report the average value of $|L_{(3)}|$ as a function of β for the different quark masses explored. In most cases the average modulus is small at low β (confined) phase; then starts growing at a critical value $\beta_{c/d}$ separating the confined from the deconfined phase; a second critical value $\beta_{d/s}$ is reached, separating the deconfined from the split (skewed) phase, at which the modulus drops roughly by a factor 3; finally a third critical value $\beta_{s/r}$ is met where the system goes into the new center symmetric phase with a small average modulus for $L_{(3)}$. In the case of $am = 0.5$ only two phases (confined and deconfined) are visible in figure 3, simply because of the limited range of β shown in the figure. The evidence for the existence

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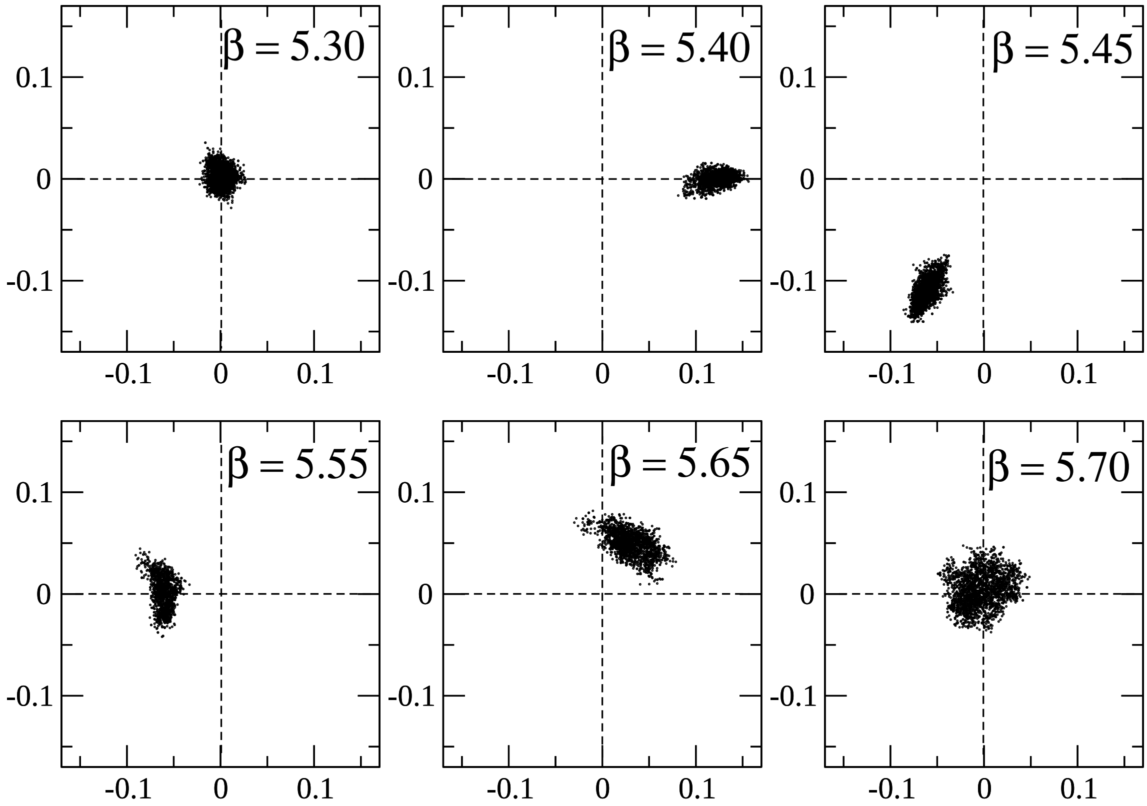

Figure 2. Same as in figure 1, but for $am = 0.02$.

of a split (skewed) phase is weaker at the smallest quark mass explored, $am = 0.01$. Typical time histories for the real and imaginary parts of $L_{(3)}$ are shown in figure 4, in particular for the confined and deconfined phase at the lowest quark mass explored, $am = 0.01$.

It is interesting to notice that in each different phase the Polyakov line modulus has a very small dependence on the quark mass, much weaker than what observed in the case of finite temperature QCD(Adj) (see for instance results reported in ref. [14, 15]). In the present case the main effect of changing the quark mass seems that of moving the location of the phase transitions.

Locations of the critical couplings as a function of the bare quark mass are reported in table 1. Regarding the order of the phase transitions, we have indications, coming from abrupt jumps of observables or presence of metastabilities, that they are all first order, at least for $am \geq 0.05$. For the smallest quark masses, $am = 0.01$ and $am = 0.02$, transitions look smoother. A deeper investigation and a finite size scaling analysis would be needed to reach a definite conclusion: that goes beyond our present purposes. However we notice that true phase transitions (continuous or discontinuous) should be found at points where the realization of the (exact) center symmetry changes.

Let us now discuss our results. The confining behaviour observed at large L_c (low β) is in agreement with the usual infinite volume behaviour of QCD(Adj). The confining behaviour observed at small L_c (high β) is in agreement with the prediction of the one-loop

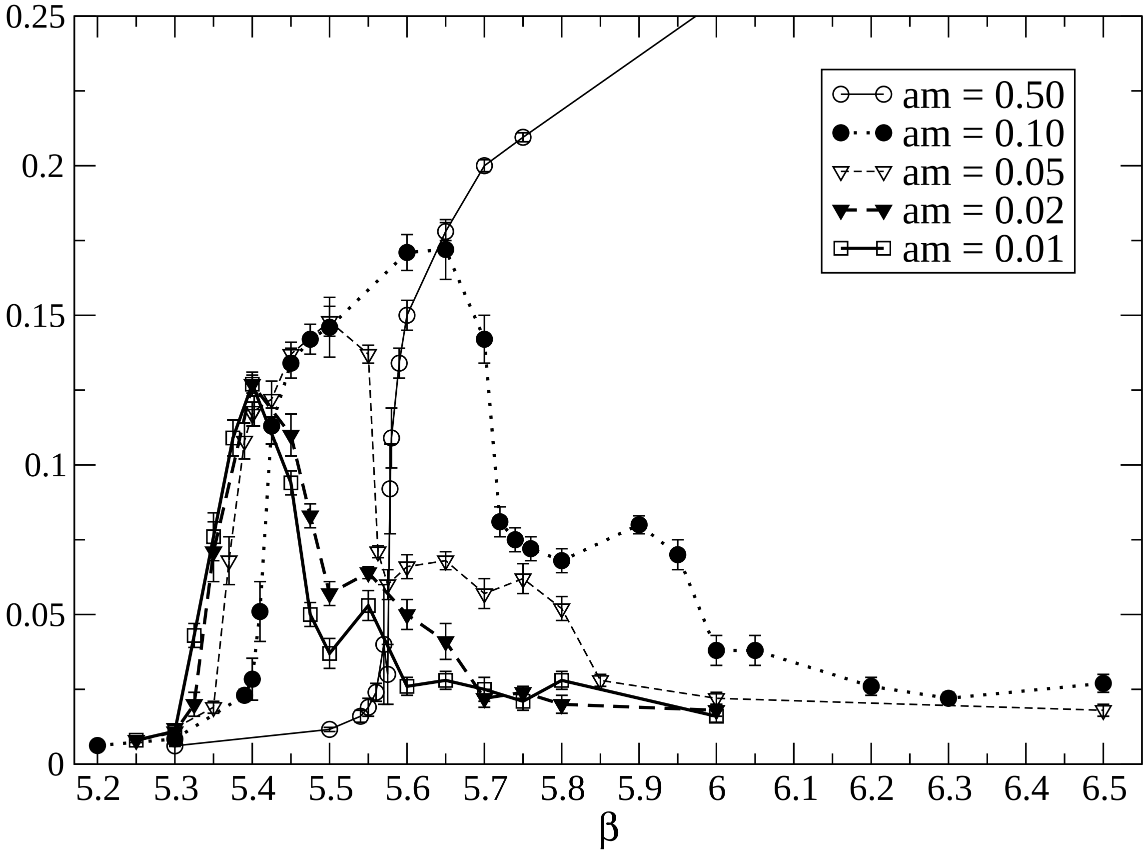

Figure 3. Average value of the Polyakov line modulus, on a $16^3 \times 4$ lattice, as a function of β and for different bare quark masses.

am	$\beta_{\mathrm{c/d}}$	$\beta_{\mathrm{d/s}}$	$\beta_{s/c}$
0.50	5.575(5)	7.50(10)	8.50(20)
0.10	5.41(1)	5.71(1)	6.00(5)
0.05	5.37(1)	5.56(1)	5.82(3)
0.02	5.33(1)	5.485(10)	5.65(3)
0.01	5.32(1)	5.465(10)	5.55(5)

Table 1. Critical values of β as a function of the bare quark mass for the transition from the confined to the deconfined phase ($\beta_{\mathrm{c/d}}$), from the deconfined to the split (skewed) phase ($\beta_{\mathrm{d/s}}$), and from the split to the weak coupling confined (re-confined) phase ($\beta_{s/c}$). In the case of $am = 0.01$ the split phase is absent and $\beta_{\mathrm{d/s}}$ actually indicates the transition from the deconfined to the re-confined phase.

effective potential in eq. (1.1): fermions contribute with a repulsive, disordering interaction for the Polyakov line eigenvalues, which is opposite and wins over the ordering gluon contribution. What is non-trivial is the behaviour observed at intermediate values of L_c, where it is clear that higher order and finite quark mass corrections to the one-loop effective potential play a significant role. In the deconfined phase those corrections result in an

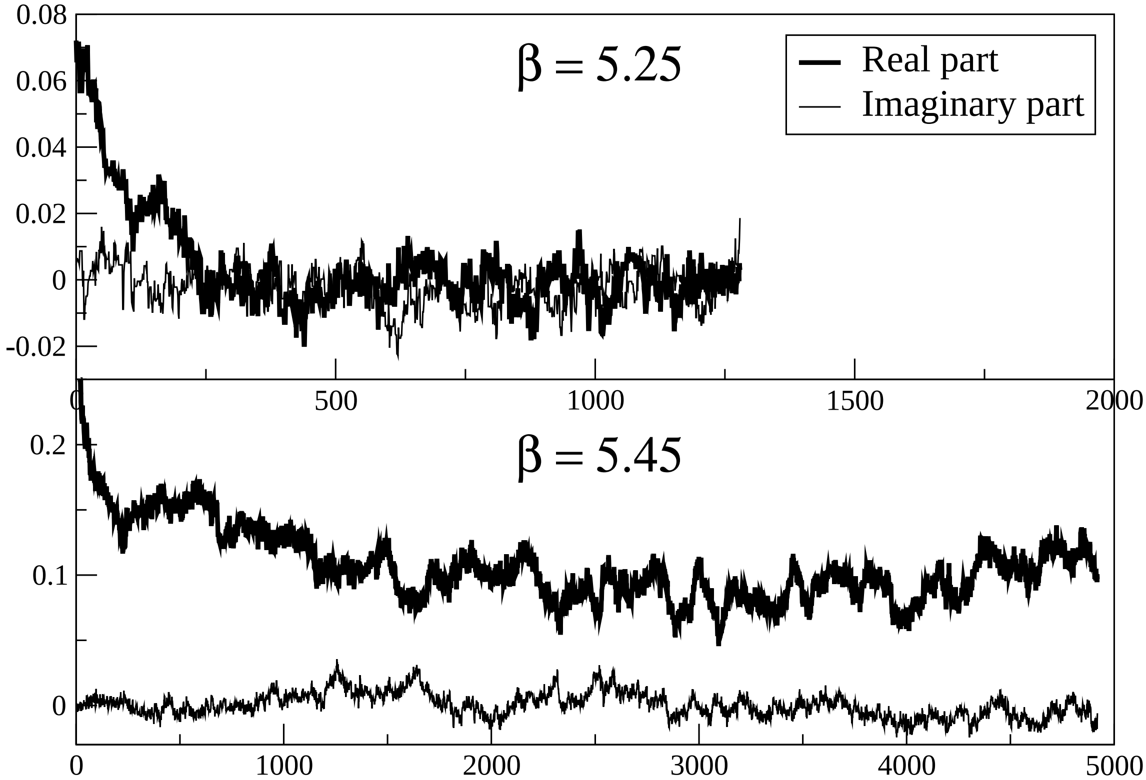

Figure 4. Time histories (in units of Molecular Dynamics trajectories and including part of the thermalization) of the real and imaginary part of $L_{(3)}$, for $am = 0.01$ and two different β values in the confined and deconfined phase respectively.

overall attractive interaction for the eigenvalues which orders the Polyakov line along a center element. In the case of the split (skewed) phase instead, the free energy minimum is apparently reached as two eigenvalues coincide and the third one gets a phase π with respect to them, as in the following SU(3) matrix

$$e^{ik/N_c} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.1}$$

That is not a minimum for the potential in eq. (1.1), which is instead minimized (in the case of p.b.c. and for $N_f > 1$) by the traceless matrix $\mathrm{diag}(1, e^{2i\pi/3}, e^{-2i\pi/3})$, however it has been verified in ref. [10] that mass corrections to the 1-loop potential may move the minimum to group elements like that in eq. (3.1). The trace of such group elements is 1/3 with respect to that of center elements: that roughly explains the drop observed for the modulus of $L_{(3)}$ at $\beta_{d/s}$.

It is also interesting to look at the behaviour of the adjoint Polyakov loop, which is shown in figure 5 for the case $am = 0.05$: it is positive only in the deconfined phase and negative outside. Since $\mathrm{Tr}_{(8)} = |\mathrm{Tr}_{(3)}|^2 - 1$, the large negative values obtained for the average adjoint Polyakov loop in the re-confined phase can be interpreted as the fundamental

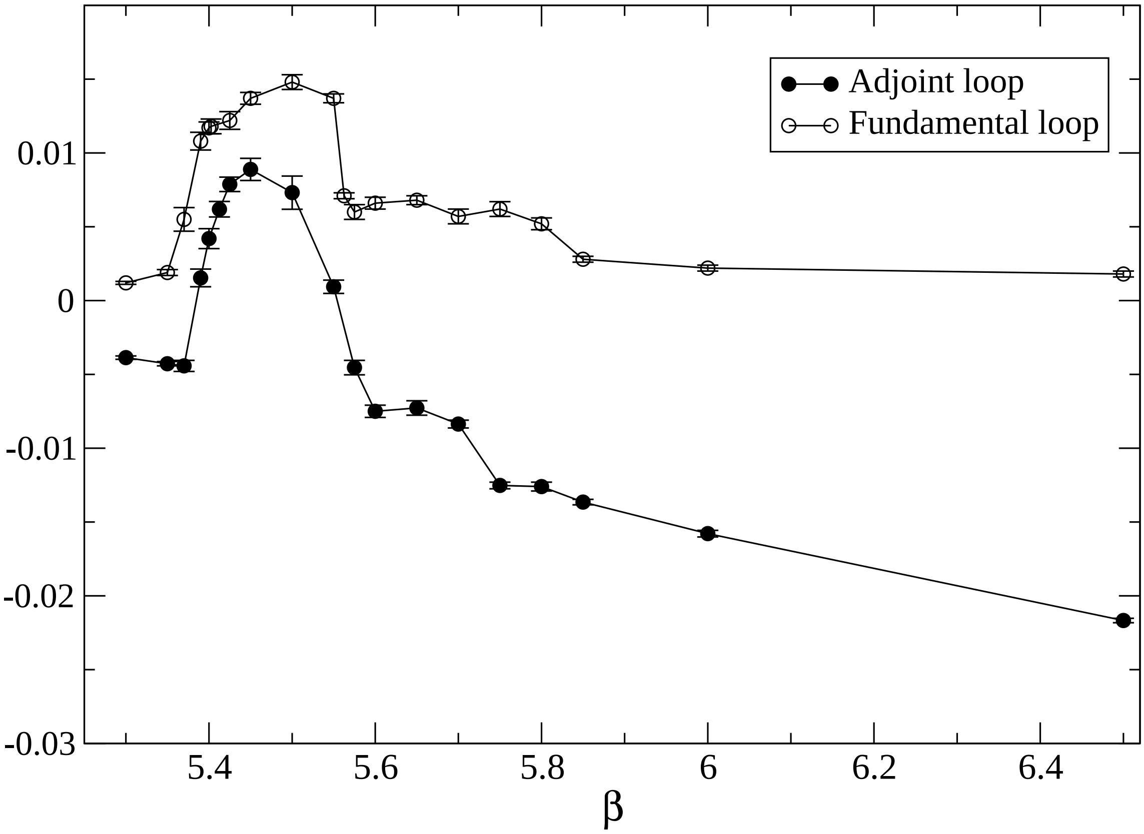

Figure 5. Adjoint Polyakov loop compared to the modulus of the fundamental loop (divided by a factor ten to fit in the figure), as a function of β for $am = 0.05$ on a $16^3 \times 4$ lattice.

loop becoming traceless point by point in the weak-coupling limit, in agreement with the 1-loop potential in eq. (1.1), while in the strong coupling regime the fundamental trace averages to zero mainly because of long range disorder.

In the case of bare quark mass $am = 0.10$ we have repeated our runs on a $16^3 \times 6$ lattice, i.e. taking $\hat{L}_c = 6$ sites in the compactified direction. Results for the modulus of $L_{(3)}$ are reported in figure 6, where they are compared with results obtained on the $16^3 \times 4$ lattice. The phase structure does not change and all the critical couplings move to larger values: that is consistent with the possible existence of a continuum limit for the critical values of the compactified dimension $L_c = \hat{L}_c a(\beta, am)$. In particular the deconfinement critical coupling moves from $\beta_{c/d}(\hat{L}_c = 4) = 5.41(1)$ to $\beta_{c/d}(\hat{L}_c = 6) = 5.50(1)$: that is consistent with a constant critical length $L_{c/d} = \hat{L}_c a(\beta_{c/d})$, since using the 2-loop β-function

$$a\Lambda_L \approx R(\beta) = \left(\frac{\beta}{6b_0}\right)^{b_1/2b_0^2} \exp\left(-\frac{\beta}{12b_0}\right) \tag{3.2}$$

with b_0 and b_1 given by [18]

$$b_0 = \frac{3}{16\pi^2}\,, \quad b_1 = -\frac{90}{(16\pi^2)^2}\,, \tag{3.3}$$

we get

$$\frac{a(\beta_{c/d}(\hat{L}_c = 4))}{a(\beta_{c/d}(\hat{L}_c = 6)} \simeq 1.6(2) \ .$$

$\beta_{d/s}$ and $\beta_{s/r}$ show a larger change when going from $\hat{L}_c = 4$ to $\hat{L}_c = 6$, which cannot be interpreted in terms of the 2-loop β-function. This is due to the fact that we are not working at fixed physical quark mass, but instead at fixed am, meaning larger and larger m as β increases: as we increase $\hat{L}_c$ we have to increase β to make a smaller and keep $L_c = a\hat{L}_c$ fixed, but at the same time, if am is fixed, also $m = (am)/a$ increases meaning that also the physical value of L_c changes. While the critical $L_{c/d}$ corresponding to deconfinement changes smoothly and stays finite as the quenched limit $m \to \infty$ is approached, so that this effect may be small, the other two transitions must disappear in the same limit, meaning a steepest dependence of the physical critical dimension on m (hence on β). We notice that for $\hat{L}_c = 6$ the transition from the deconfined to the split (skewed) phase is accompanied by strong metastabilities, testified by different simulations at equal values of β staying in different phases for thousands of trajectories (see figure 6 where the results of those simulations are reported for $\beta = 6.2$ and $\beta = 6.4$).

In order to get an idea of the scale separation between the different phases we have also performed simulations at fixed quark mass and ultraviolet (UV) cutoff, and variable $\hat{L}_c$ (only even values of $\hat{L}_c$ can be explored because of the staggered fermion discretization). In particular in figure 7 we show the distribution of $L_{(3)}$ in the complex plane and its average modulus for different value of $\hat{L}_c$ on a $16^3 \times \hat{L}_c$ lattice, at fixed bare quark mass $am = 0.1$ and inverse gauge coupling $\beta = 5.75$. The system is in the confined, center symmetric phase for $\hat{L} = 16$, where $\langle L_{(3)} \rangle = (0.0001(2), 0.0002(2))$; it is still marginally confined for $\hat{L}_c = 14$, where $\langle L_{(3)} \rangle = (0.0005(2), 0.0001(2))$, while as the compactified dimension is squeezed further a finite Polyakov line expectation value develops already for $\hat{L}_c = 12$, where $\langle L_{(3)} \rangle = (0.0066(8), -0.0008(8))$: this is clearly different from zero, even if quite small due to strong UV noise. The system stays in the deconfined phase till $\hat{L}_c = 6$, while it is clearly in the split phase for $\hat{L}_c = 4$. Finally, for the shortest compactified dimension explored, $\hat{L}_c = 2$, it is in the center symmetric, re-confined phase. We conclude that, in this case, the two center symmetric, confined phases, taking place at large or small values of the compactified dimension, are not connected: they have boundaries which are separated by a scale factor $\sim$ 3-4.

3.1 Approaching the chiral limit

Our results do not support the conjecture about volume independence of QCD(Adj), since for all explored masses the two center symmetric phases taking place for large and small values of L_c do not correspond to a unique confined phase, but are instead separated by different phases in which center symmetry is spontaneously broken. However a very important issue is whether this is true for arbitrarily small masses.

While our computational resources do not allow us to perform extensive simulations for smaller quark masses, we can try to extrapolate to the chiral limit our results for the critical values of β obtained on the $16^3 \times 4$ lattice and reported in table 1. We have therefore

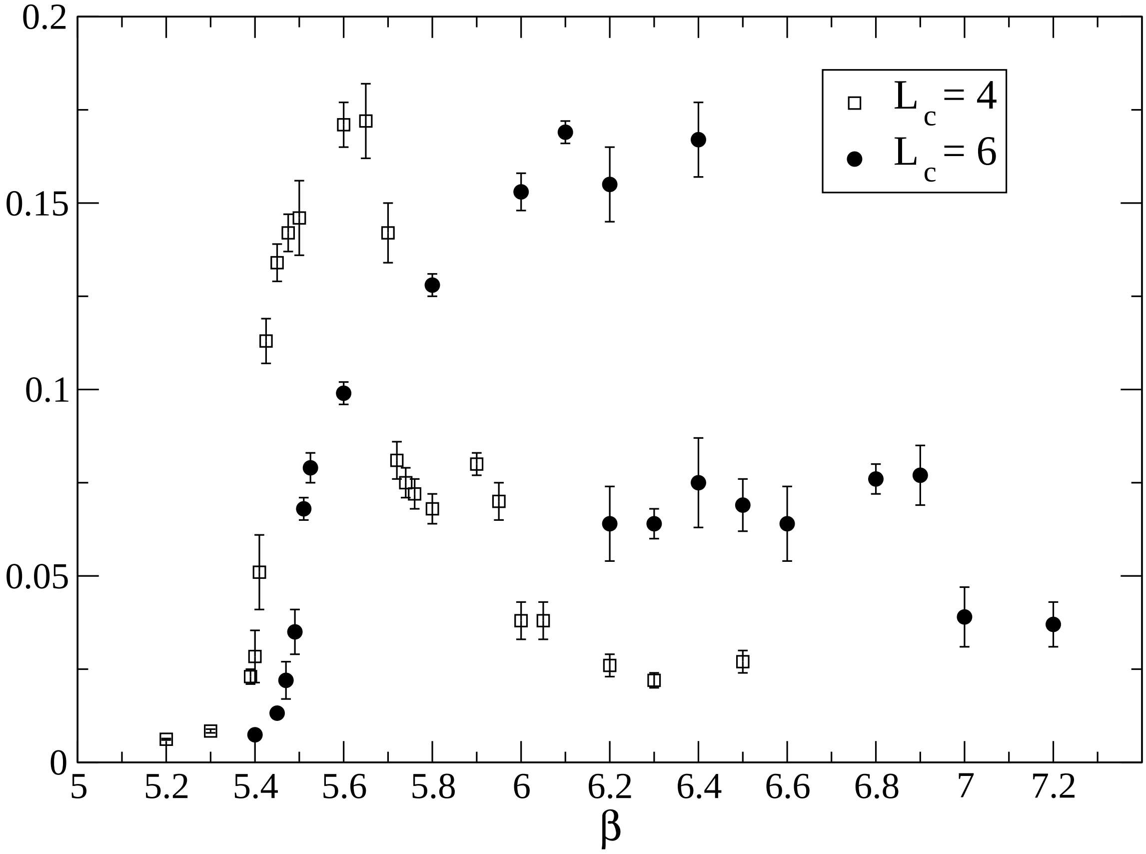

Figure 6. Comparison of results obtained for the average value of the Polyakov line modulus as a function of β at $am = 0.10$ and on two different lattices, $16^3 \times 4$ and $16^3 \times 6$.

fitted the critical values of β obtained for each different phase transition according to

$$\beta_{\mathrm{crit}} = \beta^{\chi}_{\mathrm{crit}} + c_1 am + c_2 (am)^2 \tag{3.4}$$

where $\beta^{\chi}_{\mathrm{crit}}$ indicates the critical coupling in the chiral limit.

Results are reported in figure 8. The split phase could disappear in the chiral limit, and already at $am = 0.01$ the evidence for its existence is weaker. The values of the critical couplings extrapolated to the chiral limit are the following: $\beta^{\chi}_{c/d} = 5.310(15)$ for the transition from the confined to the deconfined phase, and $\beta^{\chi}_{d/s} = 5.444(15)$ for the transition from the deconfined to the split (or re-confined) phase.

These results provide partial evidence for a deconfined phase surviving in the chiral limit. The two confined phase are not connected but still separated, even in the chiral limit, by a scale factor which is estimated, on the basis of the 2-loop β-function given in eq. (3.2), to be roughly 2. We would like to stress that this is only the result of an extrapolation to the chiral limit of the critical couplings $\beta_{c/d}$ and $\beta_{d/s}$ delimiting the deconfined phase in the phase diagram in figure 8: there is still no definite evidence that the deconfined phase actually extrapolates to $am = 0$.

In order to get more information on this issue we have performed a numerical simulation at a smaller value of the quark mass, $am = 0.005$, and for a single value of β chosen well

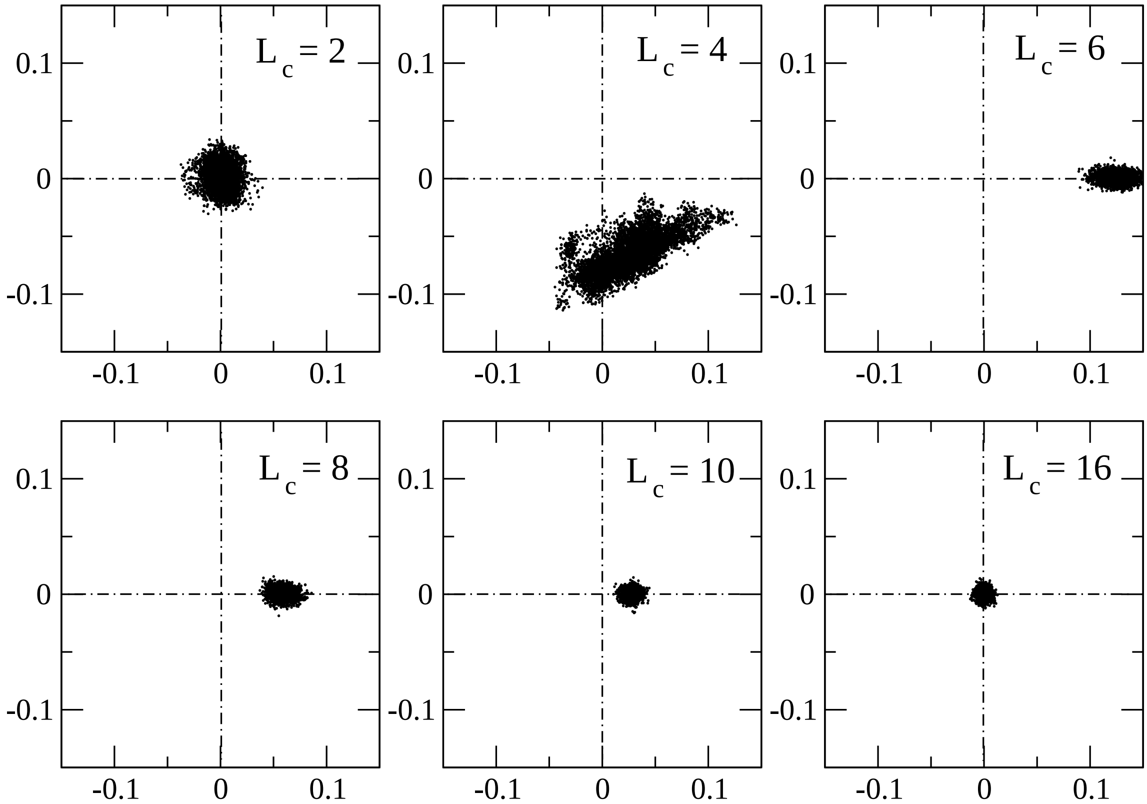

Figure 7. Scatter plots of the distribution of $L_{(3)}$ in the complex plane on a $16^3 \times \hat{L}_c$ lattice at fixed quark mass and UV cutoff ($\beta = 5.75$ at $am = 0.10$) for various values of the compactified dimension $\hat{L}_c$.

within the deconfined region in figure 8, $\beta = 5.40$. In figure 9 we report the Monte-Carlo time histories obtained for the real and imaginary part of the Polyakov loop respectively at $am = 0.01$ and $am = 0.005$. It is apparent from the figure that while for $am = 0.01$ the system stays quite stably in the deconfined phase, for the lower mass it spends part of the simulation time into a phase with a lower value of the Polyakov loop (seemingly a split phase). More extensive (not affordable) simulations would be necessary to clarify the issue, but we can at least say that some metastable behaviour is present at this mass which may suggest an even less trivial phase structure close to the chiral limit.

Regarding the continuum limit of the phase diagram sketched in figure 8, our results obtained for $\hat{L}_c = 6$ and $am = 0.1$ (see figure 6 and related discussion) show that at least the deconfinement transition line for intermediate values of the quark mass has a well defined continuum limit.

3.2 Chiral properties

It is interesting to study the fate of chiral symmetry and its connection with the different observed realizations of center symmetry in this theory. The study of finite temperature QCD(Adj) has shown that, unlike the case of ordinary QCD, phases may exist in which deconfinement is not accompanied by chiral symmetry restoration [14–16]. In the present

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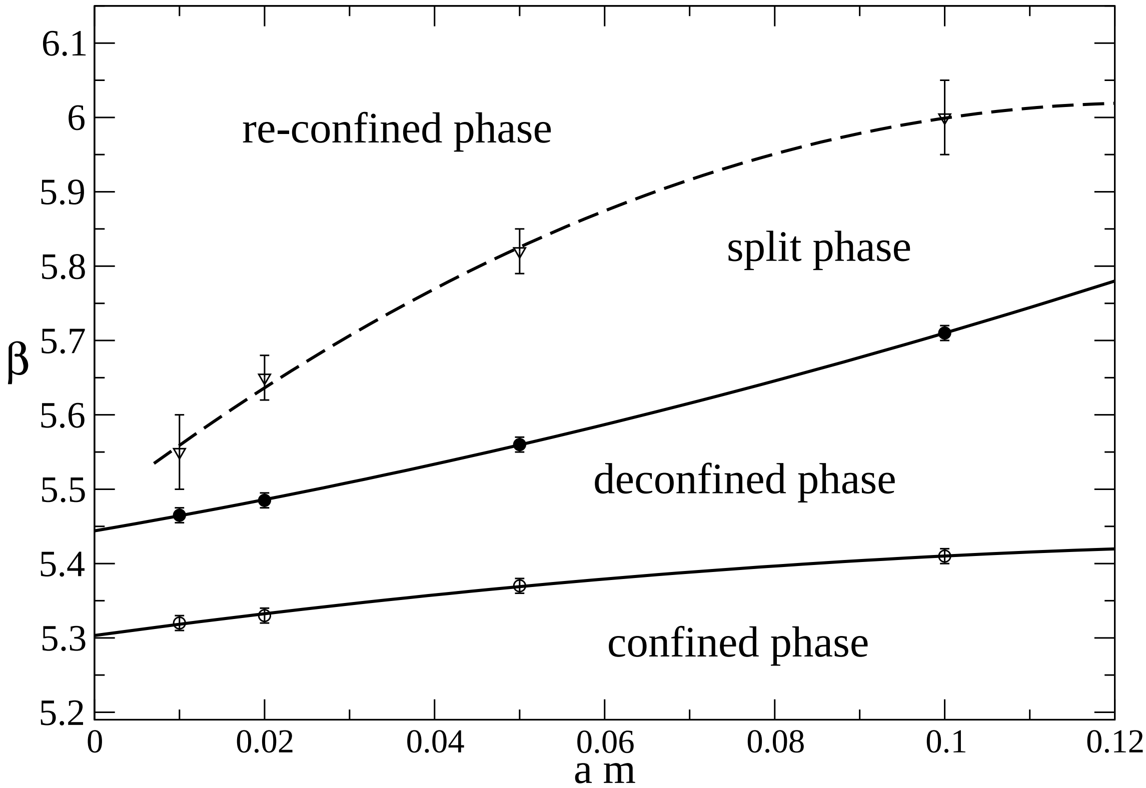

Figure 8. Possible sketch of the phase diagram in the β-am plane and approaching the chiral limit, obtained by means of quadratic interpolations of the critical couplings as in eq. (3.4).

case, since a chirally symmetric phase is expected anyway in the weak coupling regime (i.e. for short enough L_c), a new exotic phase could exist in which both chiral symmetry and center symmetry are not spontaneously broken [19, 20].

In figure 10 we show the behaviour of the chiral condensate as a function of β for different quark masses, as obtained on the $16^3 \times 4$ lattice, while in figure 11 we show the extrapolations of the same chiral condensate values to the chiral limit for different values of β which cover all the possible different phases described by the Polyakov line. Also in this case we have used quadratic extrapolations

$$\langle\bar{\psi}\psi\rangle(am) = \langle\bar{\psi}\psi\rangle(0) + a_1 am + a_2(am)^2 \tag{3.5}$$

which are shown as dashed lines in figure 11, together with similar extrapolations, including also a term proportional to $\sqrt{am}$ (continuous lines), analogous to those used in ref. [14].

Figure 11 shows that, independently of the extrapolation used, the chiral condensate always extrapolates to a non-zero value, as $am \to 0$, in all the range of β values going at least up to $\beta = 6.0$. That means that the different transitions, corresponding to different realization of center symmetry, do not affect chiral symmetry, which remains spontaneously broken.

Instead at the highest value explored, $\beta = 6.50$, there is some evidence that the chiral condensate may extrapolate to zero and chiral symmetry be restored: that would fit the

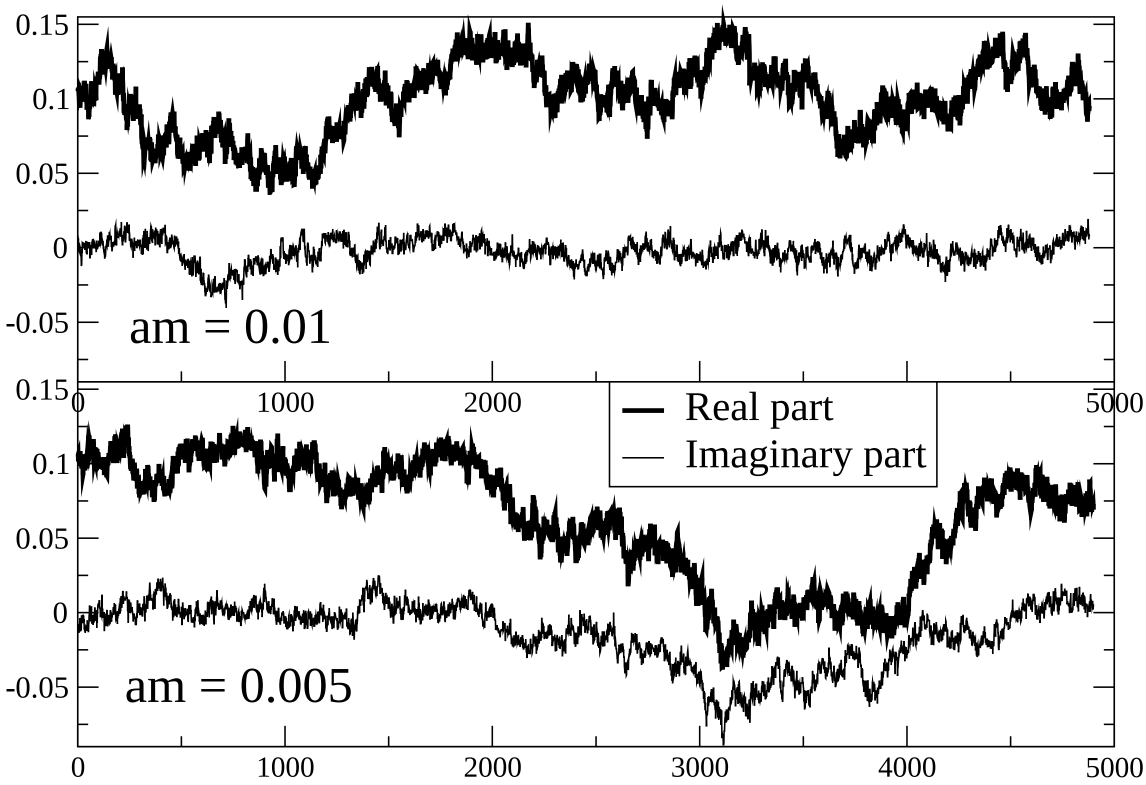

Figure 9. Time histories, in units of Molecular Dynamics trajectories, of the real and imaginary part of $L_{(3)}$, for $\beta = 5.4$ and two different values of am.

expectation of a chirally symmetric weak coupling phase. However we cannot draw a definite conclusion from present results: at this value of β the lattice spacing is very small and even $am = 0.01$ corresponds to a large quark mass, so that an extrapolation to zero quark mass may be debatable; moreover one should also properly take into account the running of the adimensional quark condensate measured on the lattice as the continuum limit $a \to 0$ is approached.

Therefore, the only reasonable conclusion that we can draw from our results is that the expected weak coupling restoration of chiral symmetry may happen at values of the compactified dimension much shorter (at least one order of magnitude, as roughly estimated on the basis of the 2-loop β-function) than those at which center symmetry transitions take place.

It is interesting to ask if the information about the change in the realization of center symmetry is transcripted in some other way into the chiral properties of the theory. A possible candidate to give an answer is the dual chiral condensate introduced in ref. [21] and defined as

$$\Sigma_1 = -\frac{1}{2\pi} \int_0^{2\pi} d\varphi \; e^{-i\varphi} \langle \bar{\psi}\psi \rangle_m^{(\varphi)} \tag{3.6}$$

where $\langle \bar{\psi}\psi \rangle_m^{(\varphi)}$ is the chiral condensate measured with an assigned phase $\exp(i\varphi)$ for the fermionic boundary conditions in the compactified dimension ($\varphi = 0$ for p.b.c. and $\varphi = \pi$

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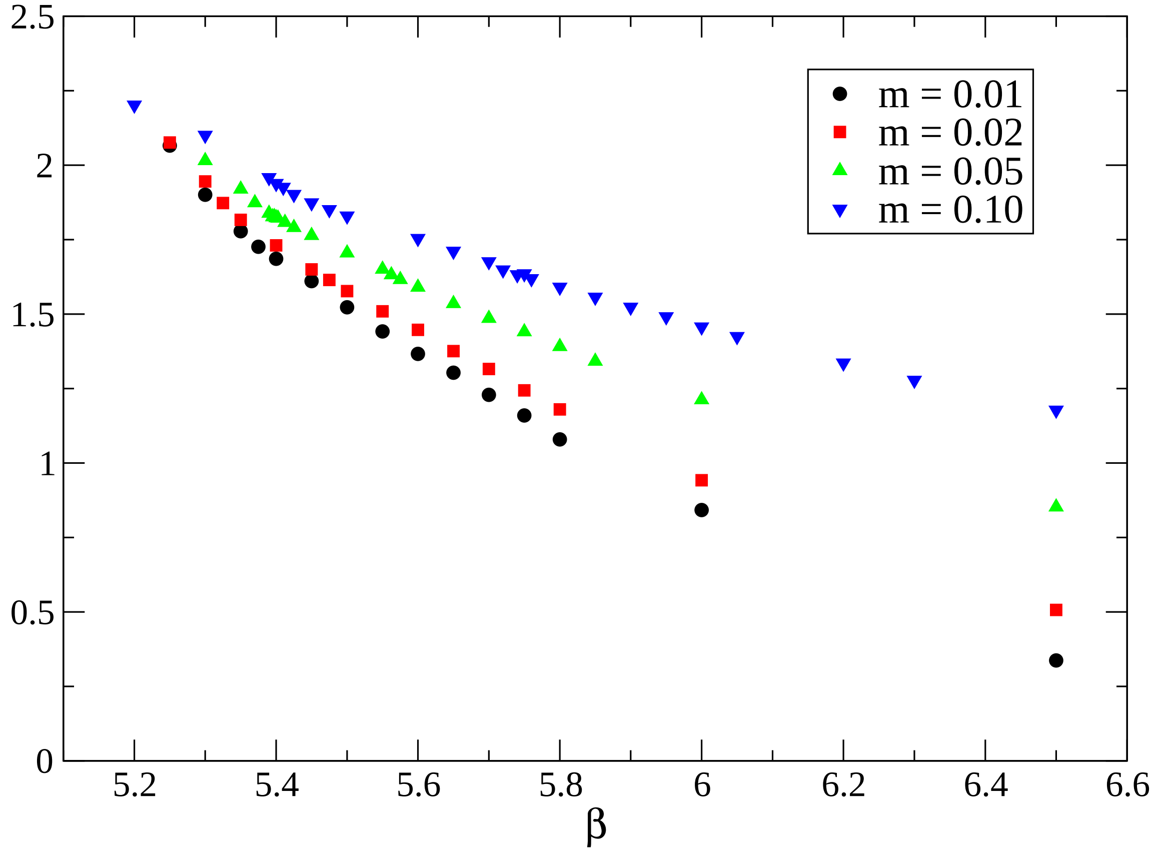

Figure 10. Chiral condensate as a function of β for different values of the bare quark mass on the $16^3 \times 4$ lattice.

for a.b.c.), and Σ_1 is nothing but its Fourier transform with respect to the phase. It can be easily shown, e.g. by a loop expansion of the fermionic determinant, that in the infinite quark mass limit the dual condensate becomes the usual Polyakov loop (adjoint Polyakov loop in this case), hence it is a natural chiral quantity which could be sensible also to the realization of center symmetry, as the adjoint loop is (see figure 5). As a very rough estimate of Σ_1, we have considered the quantity $(\langle\bar{\psi}\psi\rangle_m^{(0)} - \langle\bar{\psi}\psi\rangle_m^{(\pi)})$, i.e. the difference in the condensate measured using p.b.c. and a.b.c. respectively (that means only a change in the observable, which is measured on the same configurations produced with dynamical fermions having p.b.c.). In figure 12 we report results obtained for $am = 0.05$: as can be appreciated this difference follows quite closely the behaviour of the adjoint Polyakov loop, changing its sign in the deconfined phase. This is a confirmation, in a theory with a highly non-trivial phase structure, that the realization of center symmetry is reflected in the way the chiral condensate depends on the fermion boundary conditions [22].

4 Conclusions

We have studied QCD with three colors, 2 flavors of Dirac (staggered) fermions in the adjoint representation and one of the spatial dimensions compactified with periodic boundary

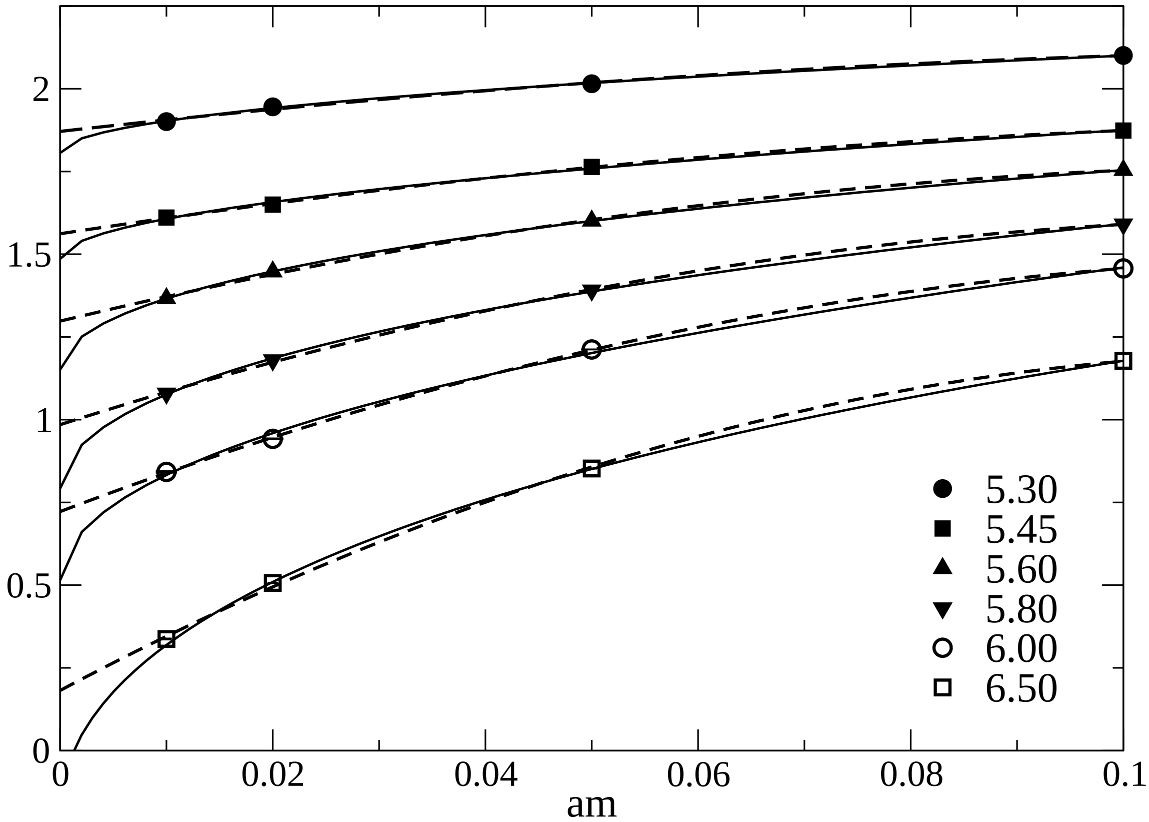

Figure 11. Chiral condensate as a function of the quark mass and quadratic extrapolations to the chiral limit including (continuous) or not including (dashed) a term proportional to $\sqrt{am}$, for different values of the gauge coupling on the $16^3 \times 4$ lattice.

conditions for fermions. We have shown that 4 different phases are explored as the size L_c of the compactified dimension is varied, corresponding to different realizations of center symmetry. In particular two center symmetric, confined phases exist, for small and large values of L_c, separated by two phases (deconfined and split phase) in which center symmetry is spontaneously broken in two different ways. We have some partial evidence that the deconfined phase could persist and separate the two confined phases also in the chiral limit. This conclusion is based on an extrapolation to the chiral limit of the critical lines determined for quark masses down to $am = 0.01$. On the other hand a direct simulation at a lower mass, $am = 0.005$, and at $\beta = 5.4$, which is inside the deconfined phase according to the extrapolations above, has shown some sign of metastability which could signal a less trivial phase structure close to the chiral limit. More extensive simulations at lower masses, which are not affordable by our present computational resources, would be needed to better clarify the issue.

Regarding the continuum limit, our results obtained for different values of the lattice spacing (see figure 6) show that the deconfinement transition that breaks center symmetry has a well defined continuum limit, at least for intermediate values of the quark mass. More careful studies would be needed to investigate the continuum limit of the other phase

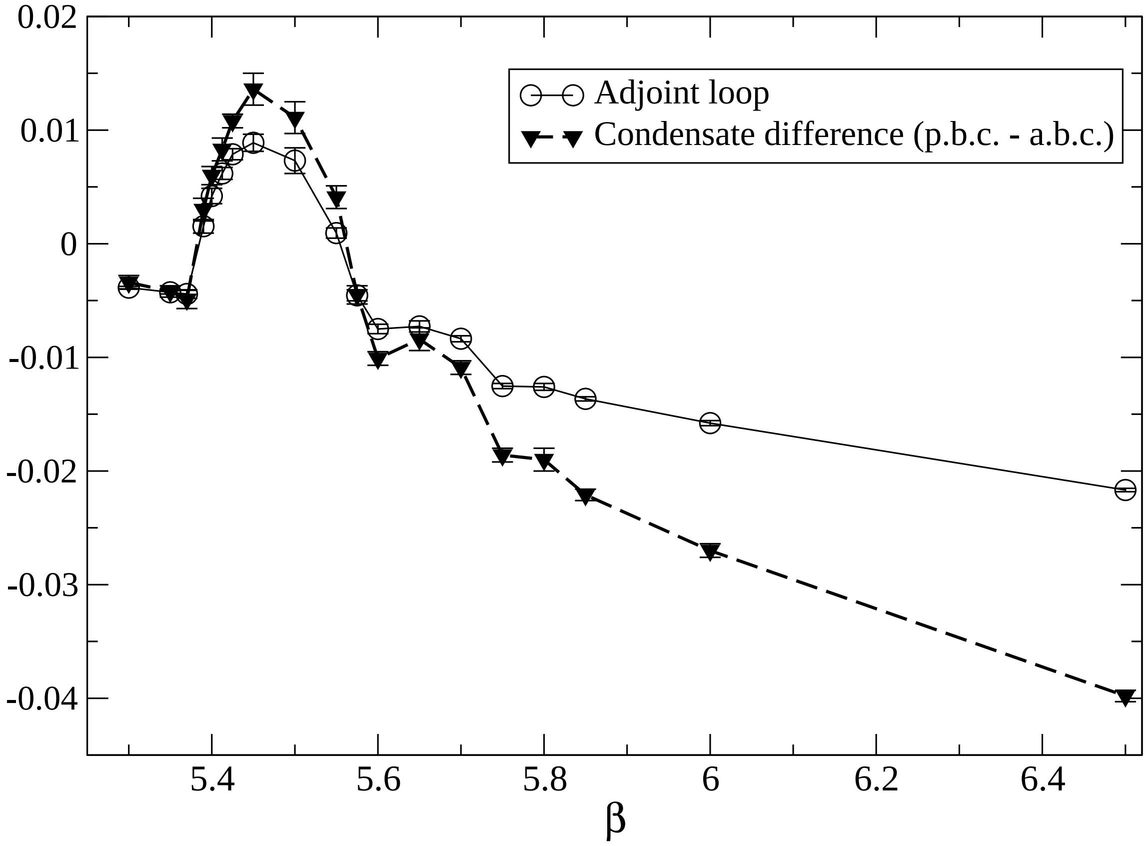

Figure 12. Comparison between $(\langle\bar{\psi}\psi\rangle_m^{(0)} - \langle\bar{\psi}\psi\rangle_m^{(\pi)})$ (see text) and the adjoint Polyakov loop for $am = 0.05$ on a $16^3 \times 4$ lattice.

transitions. However, the mere existence of a center symmetry breaking transition and of a center symmetric phase for asymptotically small values of the compactified dimension should imply the existence of other transitions also in the continuum limit.

We have investigated the chiral properties of the theory and have provided evidence that chiral symmetry is not restored until very small values of L_c, at least one order of magnitude shorter than those at which center symmetry transitions take place. On the other hand, we have verified that the realization of center symmetry is reflected in the way the chiral condensate depends on the fermion boundary conditions, thus confirming similar results obtained in ref. [22].

Our results may be relevant for a recently proposed conjecture about volume independence of QCD(Adj) in the large N_c limit [7]: in particular the presence of a deconfined phase separating the large and small volume confined phases would not support such conjecture. We have shown that this is indeed the case for $N_c = 3$, therefore this possibility should be taken into account also for larger values of N_c. Of course, apart from the fact that the extrapolation to the chiral limit of our findings should be further investigated, our results could be relevant just to the particular case studied here and to the discretization setup used. It is therefore essential to perform further studies. In particular one should verify our results as the continuum limit is approached, also making use of improved actions or different discretization setups, based for instance on Wilson instead of staggered fermions.

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